How to Split a Secret into Unknown Shares

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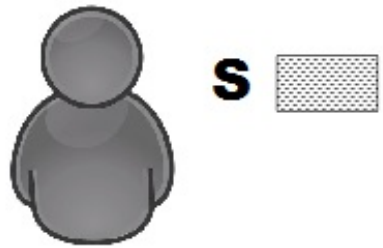
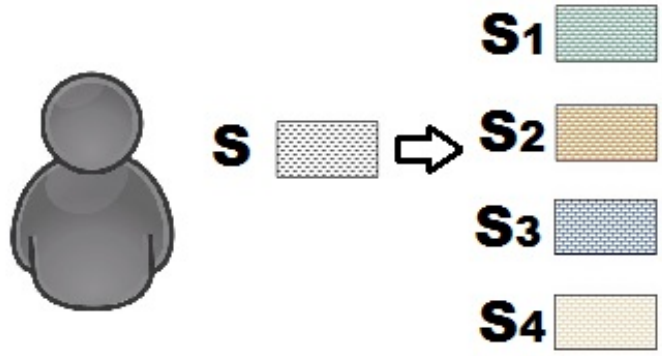
RCD - September 23, 2015

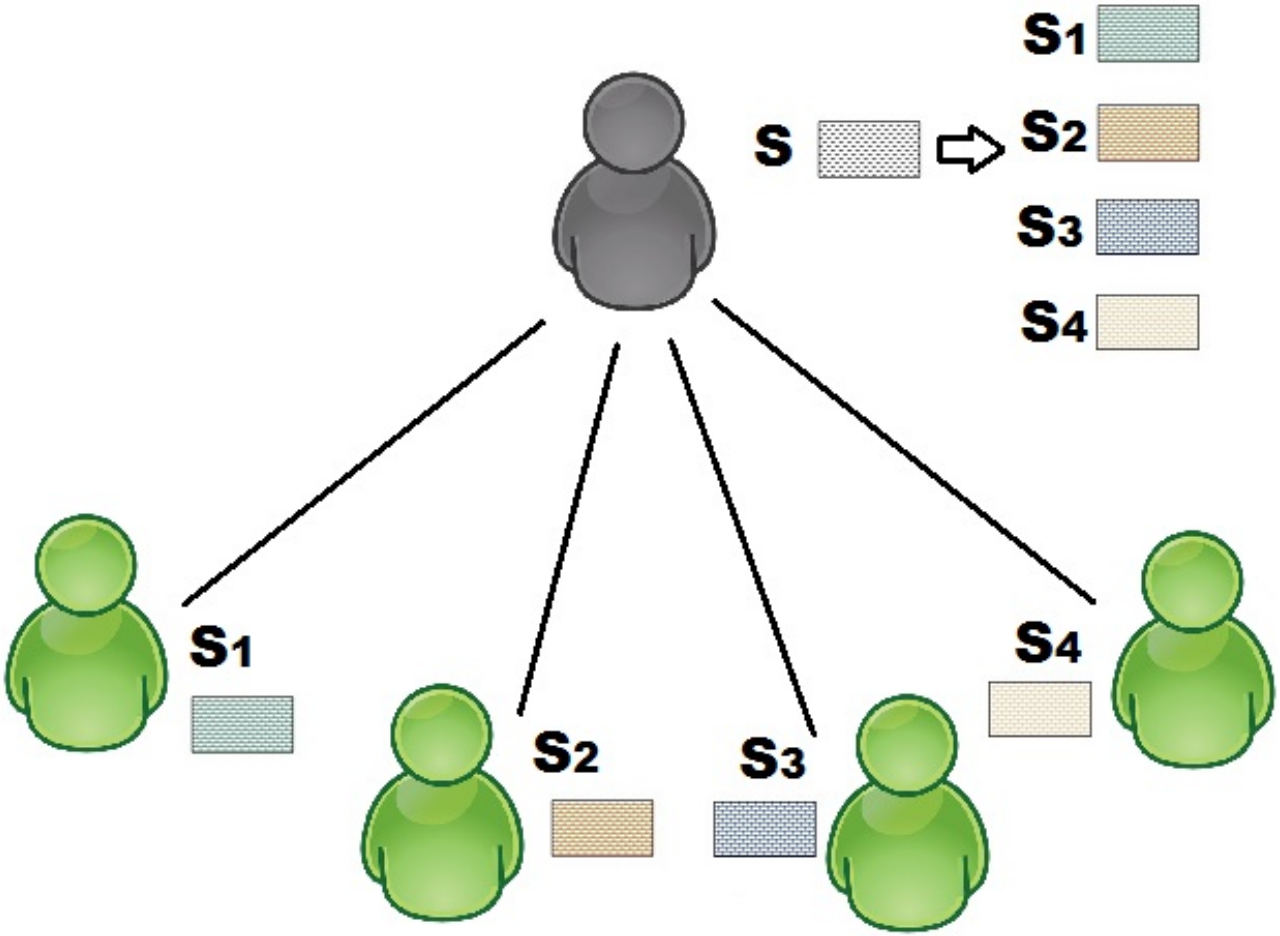
Secret Sharing



Secret Sharing



Secret Sharing



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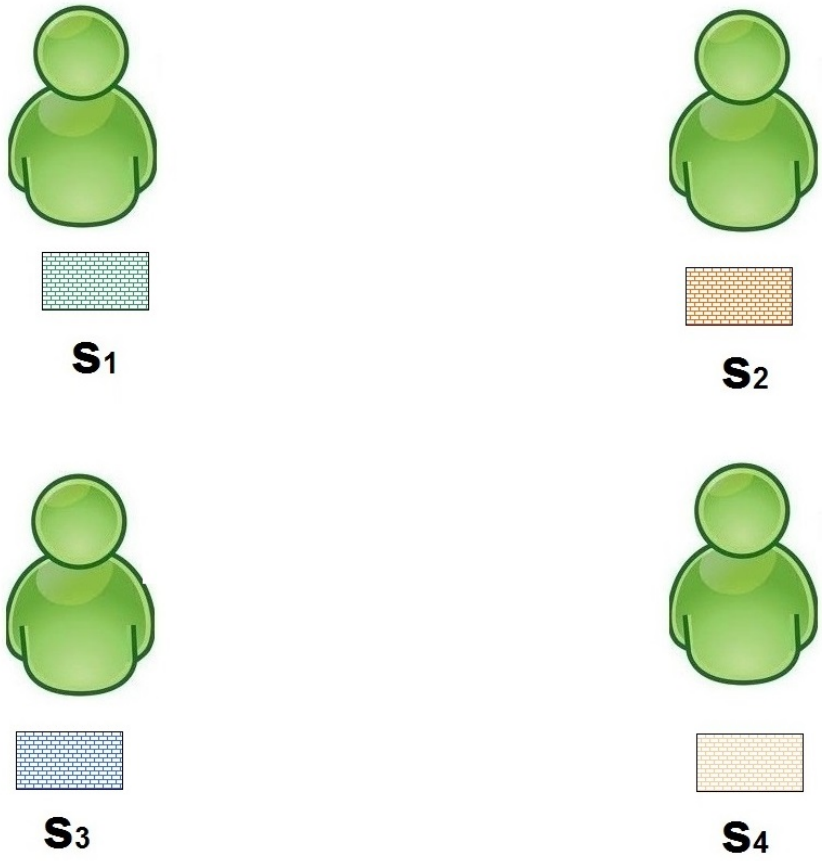

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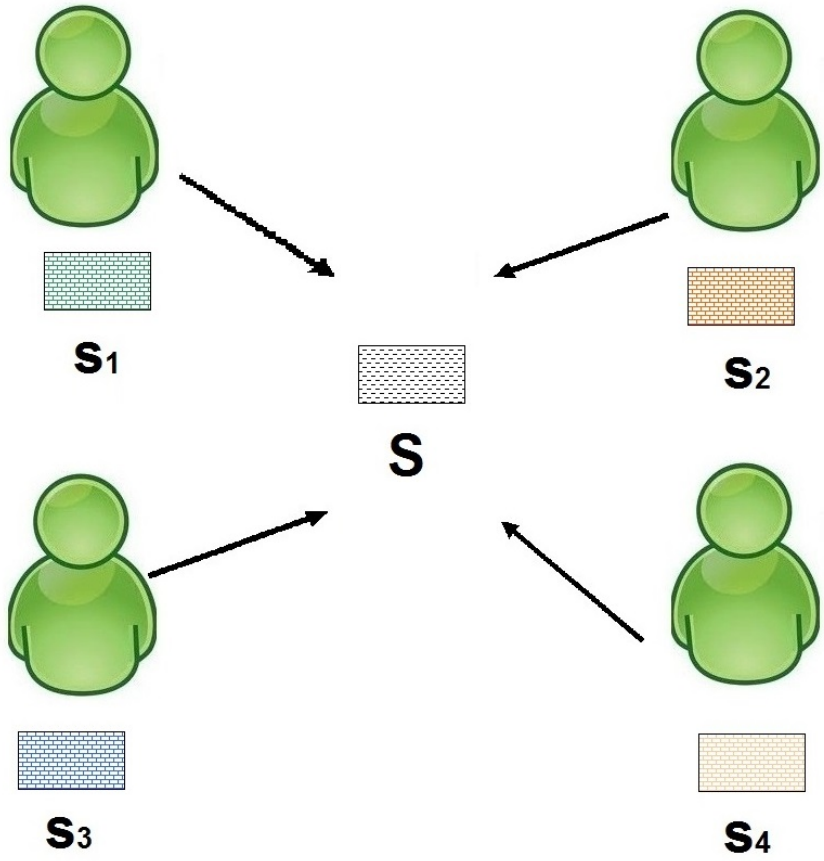

Cryptographic Anonymity

- Controversial reconstruction: it is argued if the secret should be reconstructed or not
- Coalition statistics: it is not desired to have any information about coalitions that are more likely to reconstruct

[Guillermo, Martin, O'Keefe 2003]

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What about the dealer?

Cryptographic Anonymity w.r.t. Dealer

- Existing work: all-or-nothing secret sharing

[Grigoriev, Shpilrain 2013]

- Our contribution: threshold secret sharing (Shamir's scheme)

Shamir's Secret Sharing Scheme [S'79]

1. *Secret Sharing Phase.* *The dealer picks a* $t-1$ *degree random polynomial*

$$f(x) = a_0 + a_1 x + \cdots + a_{t-1} x^{t-1} \pmod{q} \tag{1}$$

s.t. $a_0 = S$ *and* $a_i \in \mathbb{Z}_q, i = 1, \ldots, t-1$.

2. *Shares Distribution Phase.* *The dealer transmits the share* $f(x_i)$ *to the participant* U_i, $i = 1, \ldots, n$, *via a secure channel.*

3. *Reconstruction Phase.* *Given at least* t *points* $(x_i, f(x_i))$ *with distinct* x_i*'s, the unique polynomial* $f(x)$ *can be found as:*

$$f(x) = \sum_{i=1}^{t} f(x_i) \prod_{1 \le j \le t, i \ne j} \frac{x - x_j}{x_i - x_j} \tag{2}$$

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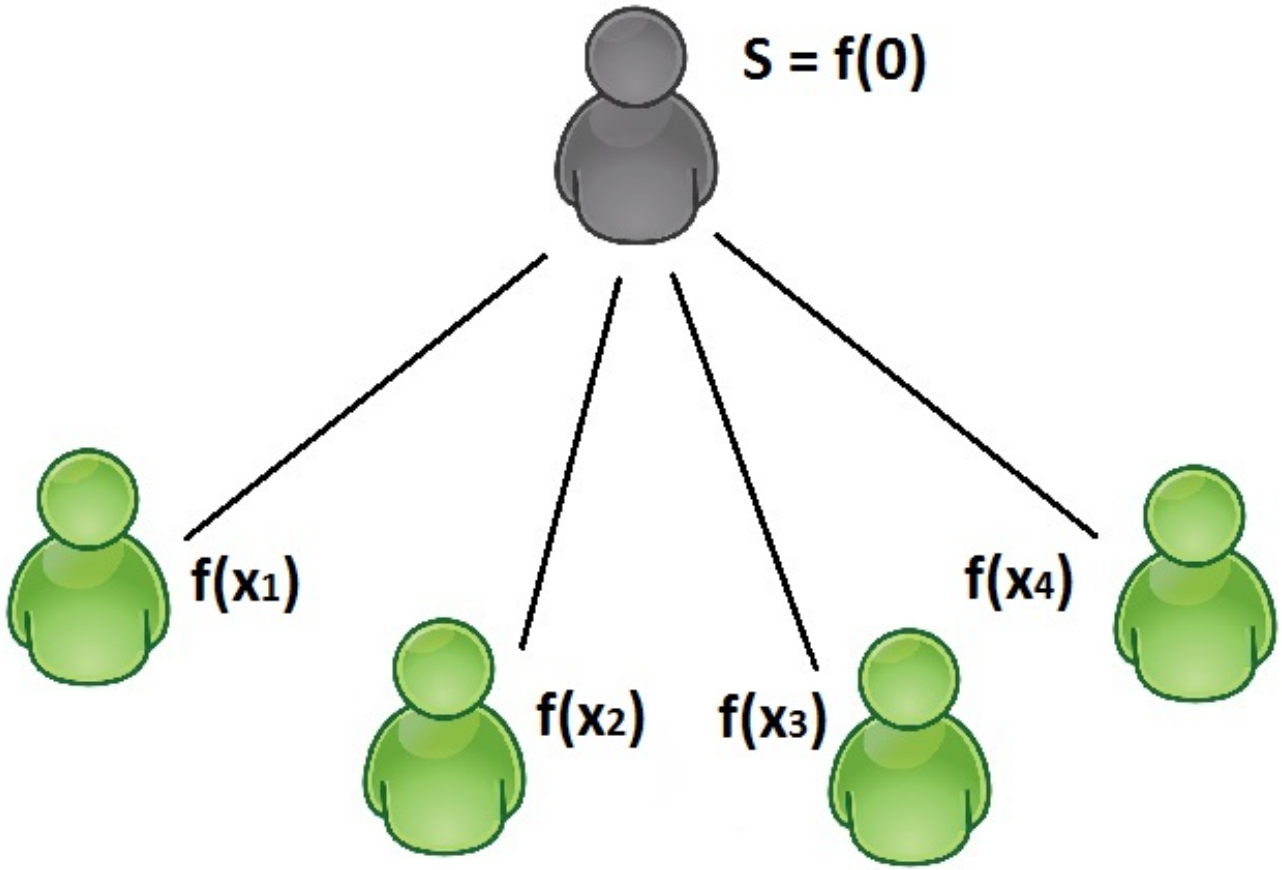

Simple Proposals

- Solution 1: the dealer *knows* the polynomial, but does *NOT know* the values where the polynomial is evaluated in
- Solution 2: the dealer does *NOT know* the polynomial, but *knows* the values where the polynomial is evaluated in

First Proposal

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Correctness and secrecy hold from construction.

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The dealer does not learn the shares (from the security of OPE)

Second Proposal

$$S = \sum_{i=1}^{n} s_i \qquad f_i(0) = s_i \qquad f(x_i) = \sum_{j=1}^{n} f_j(x_i)$$

Reconstr.: $S = \sum_{i=1}^{t+1} f(x_i) \prod_{1 \le j \le t+1, i \neq j} \frac{x_j}{x_j - x_i}$

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Correctness and secrecy hold from construction.

The dealer does not learn the final shares.

Work in progress ...

Thank you!

This work was supported by the strategic grant POSDRU/159/1.5/S/137750, Project Doctoral and Postdoctoral programs support for increased competitiveness in Exact Sciences research cofinanced by the European Social Found within the Sectorial Operational Program Human Resources Development 2007 2013.