A New Public Key Cryptosystem with Key Escrow Capabilities

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Confidentiality

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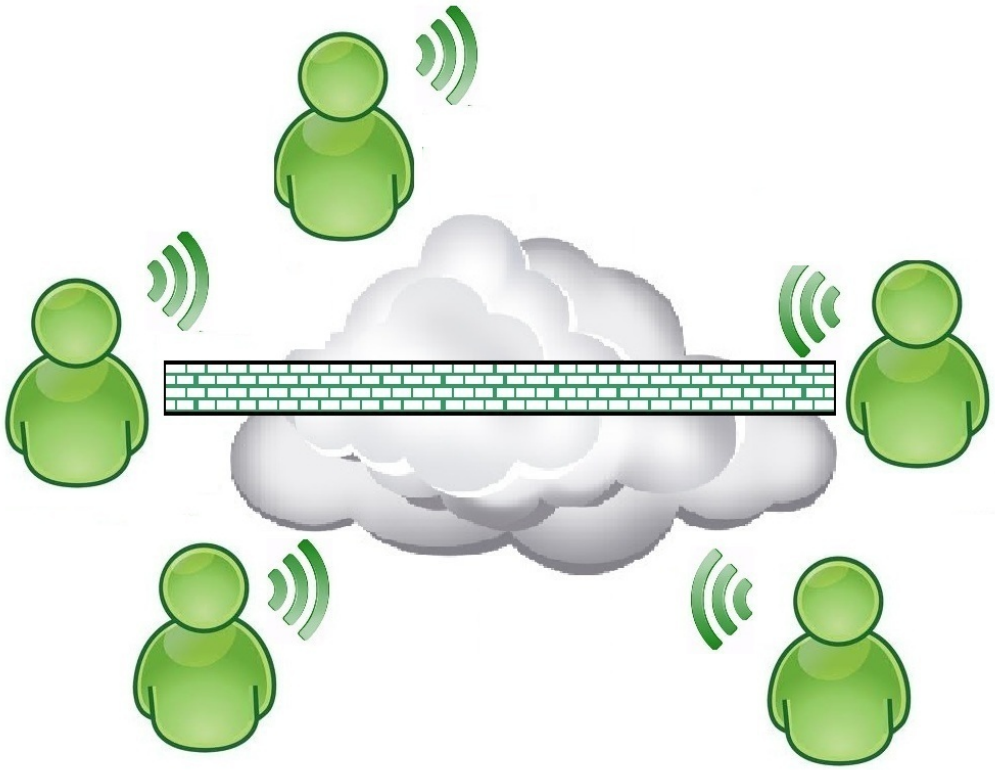
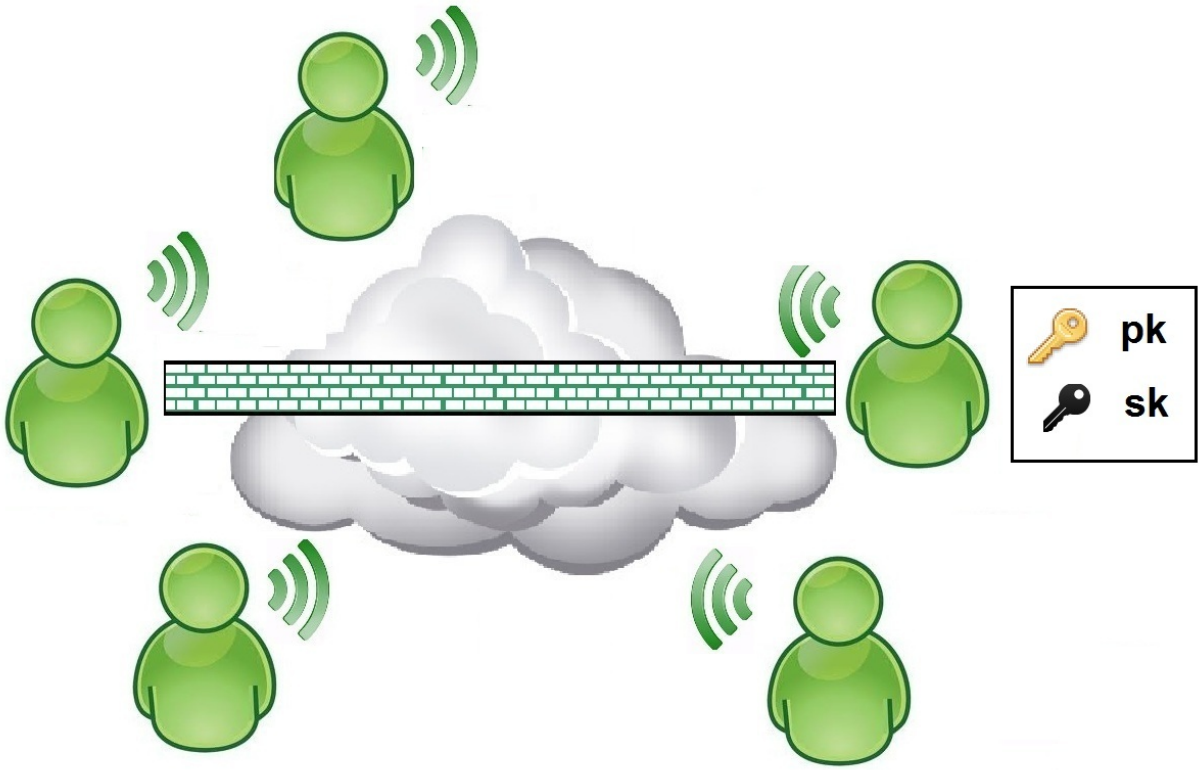


Public Key Encryption



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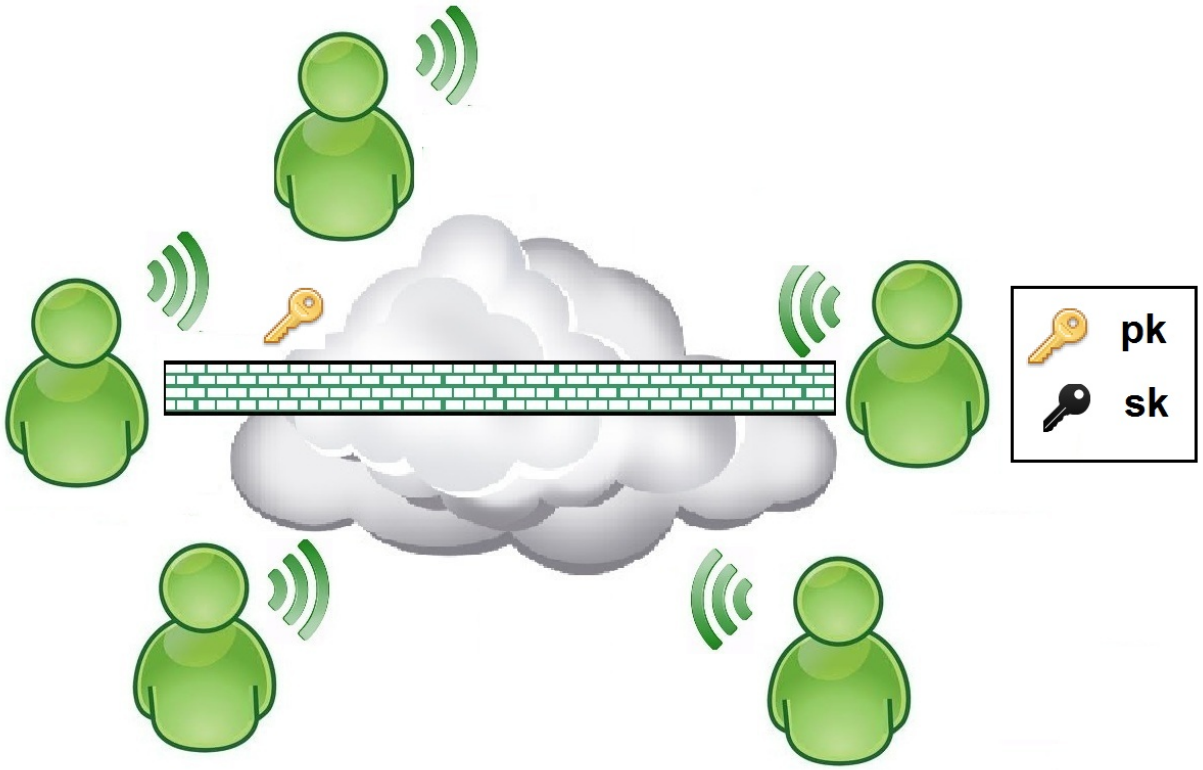
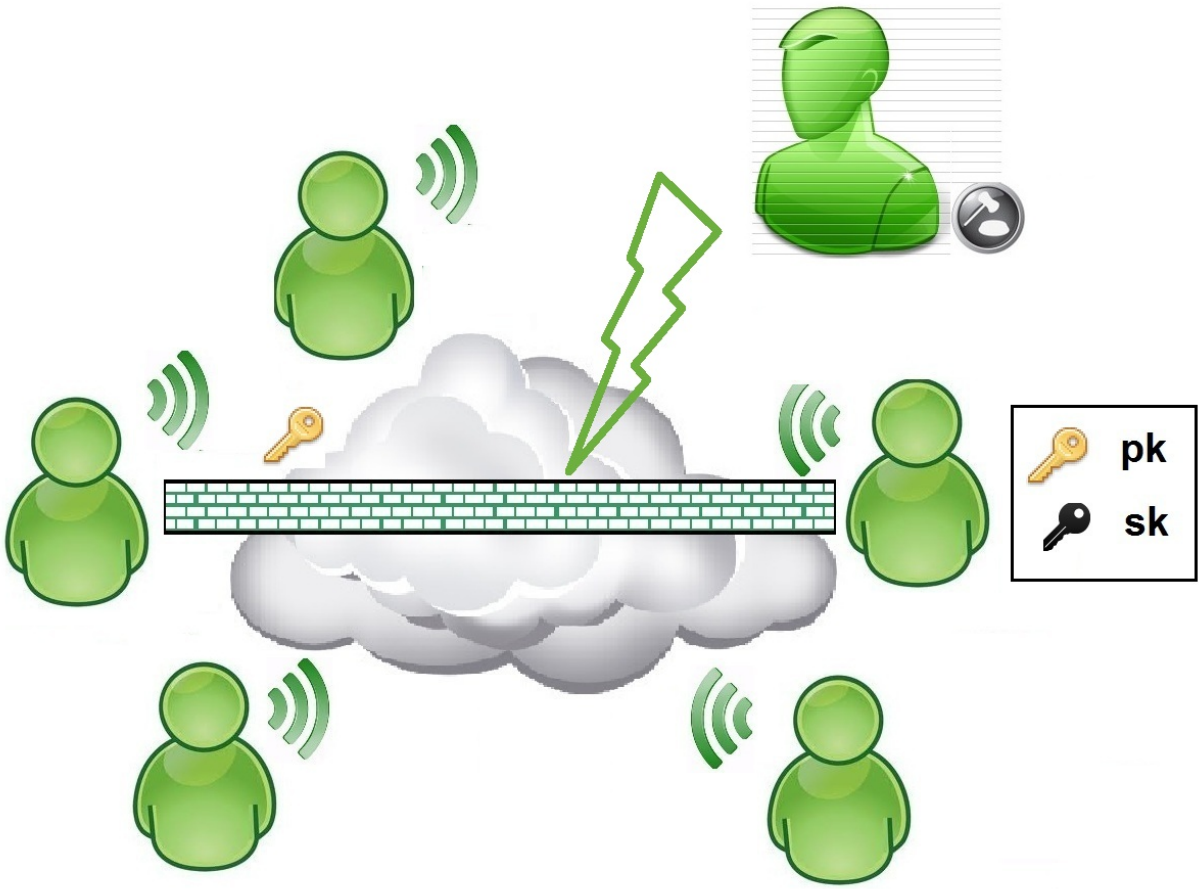


Lawful Interception



Motivation

- Communication monitoring
- Delegation of duties
- Backup of cryptographic keys

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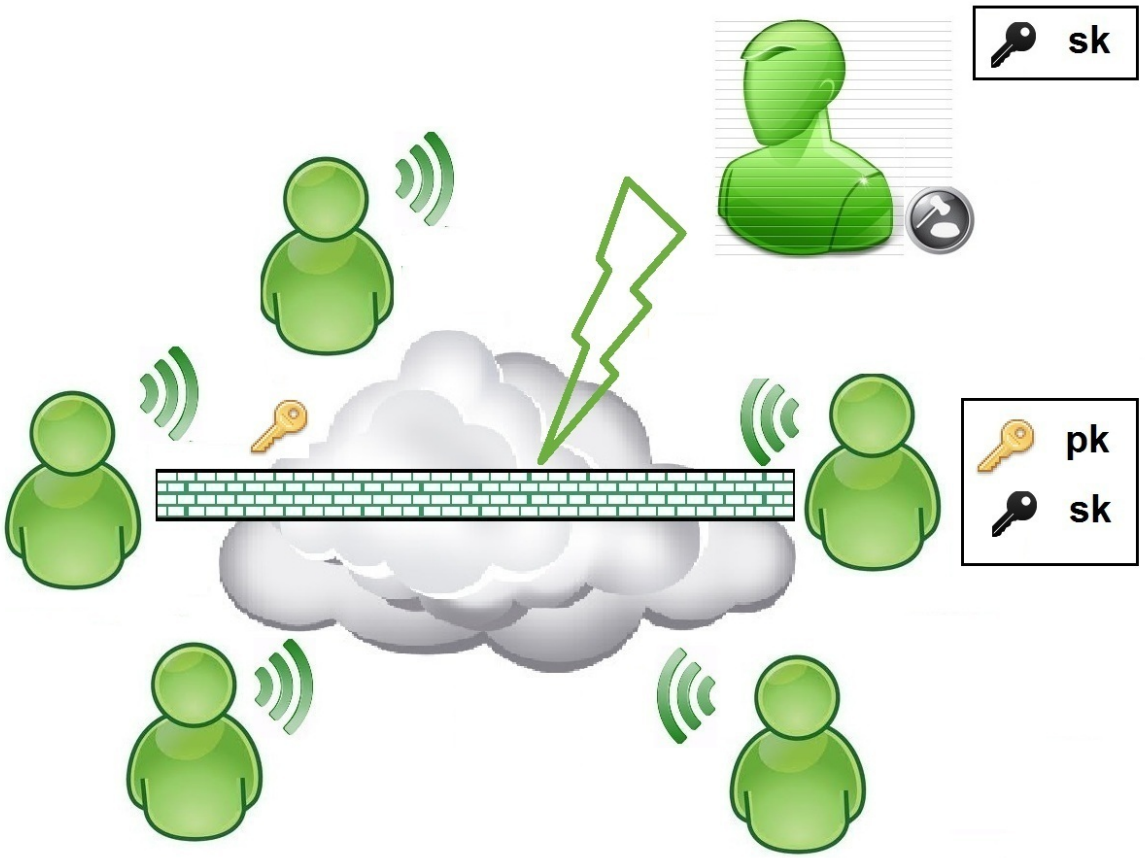
Solutions?

Trivial Approach



The authority that performs interception owns the secret keys of the users.

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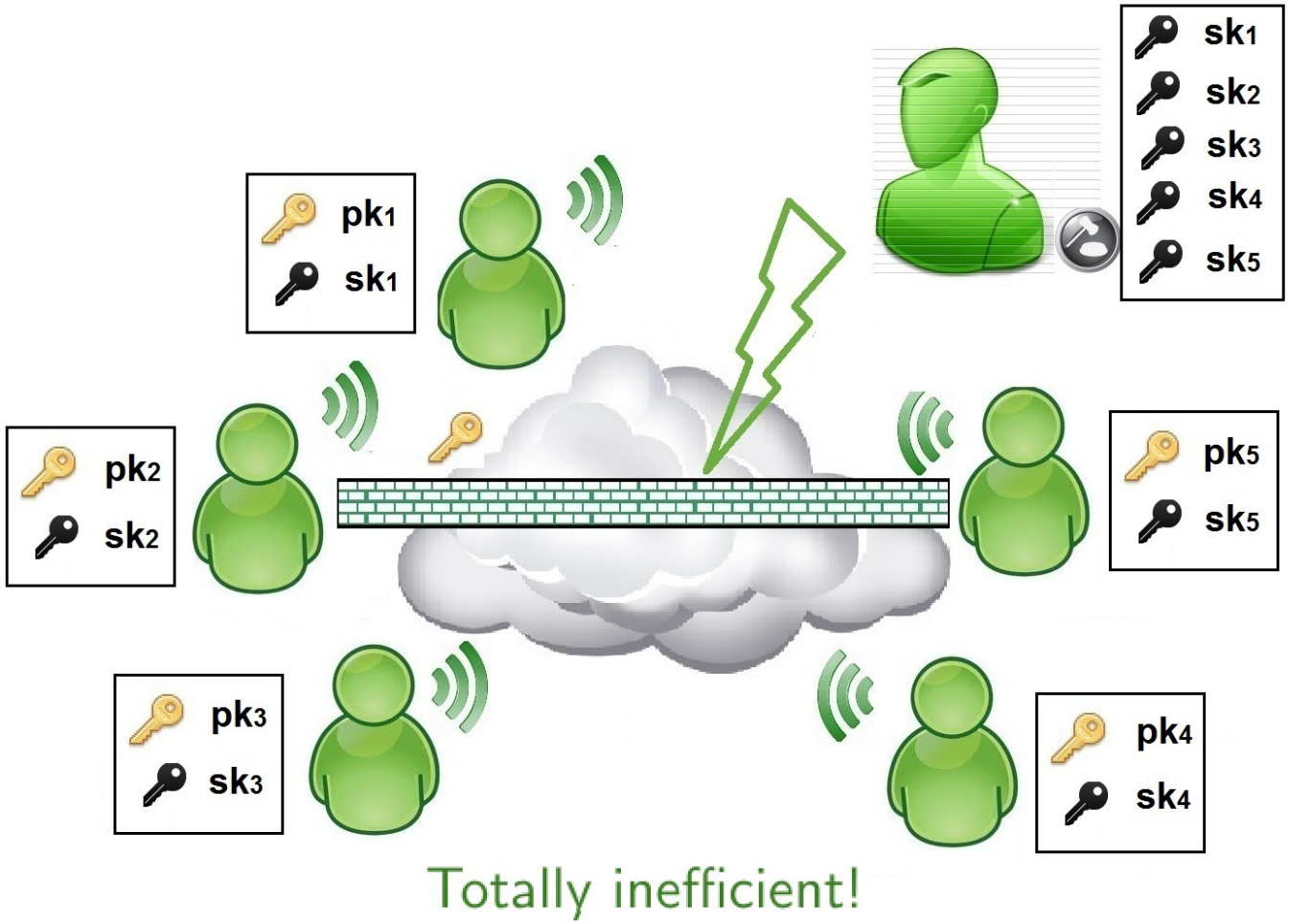

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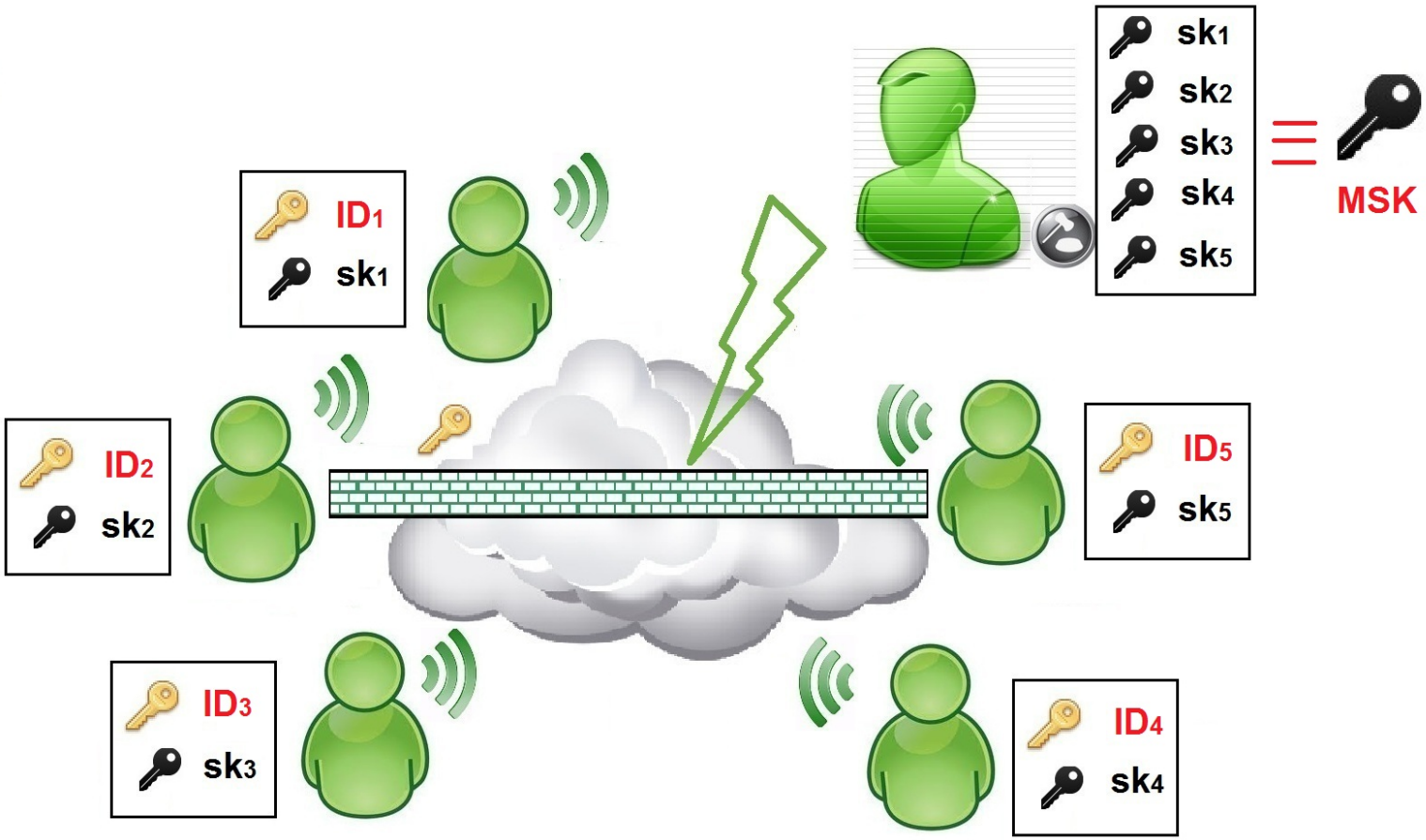
IBE (Identity Based Encryption)



Our Goal

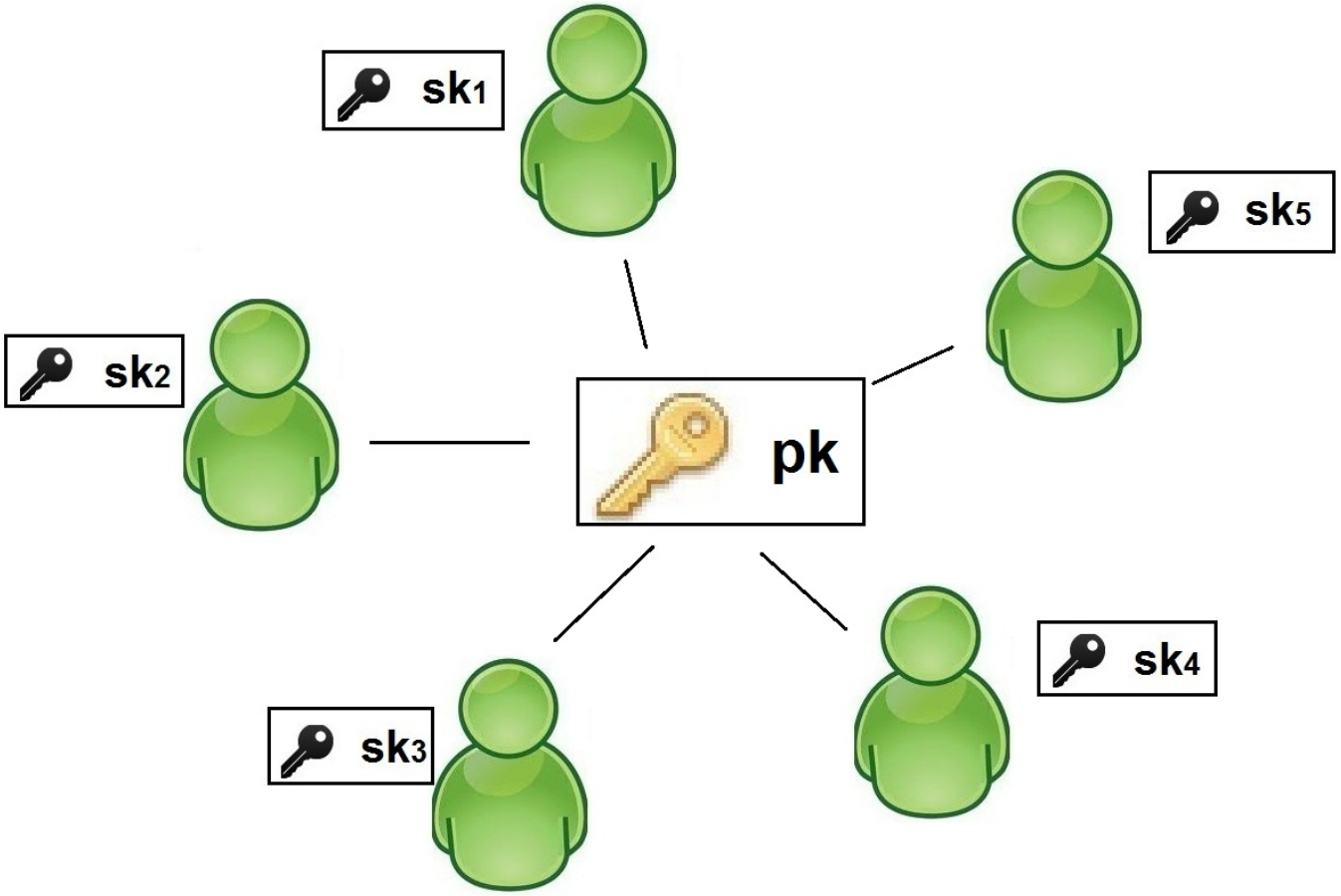
- NOT to find a better solution than IBE
- Find a solution based on ASKGA (Asymmetric Group Key Agreement)

Our Contribution

1. Define a lawful interception system based on ASGKA
2. Analyze its security and its performance
3. Extend the system to allow hierarchical structures

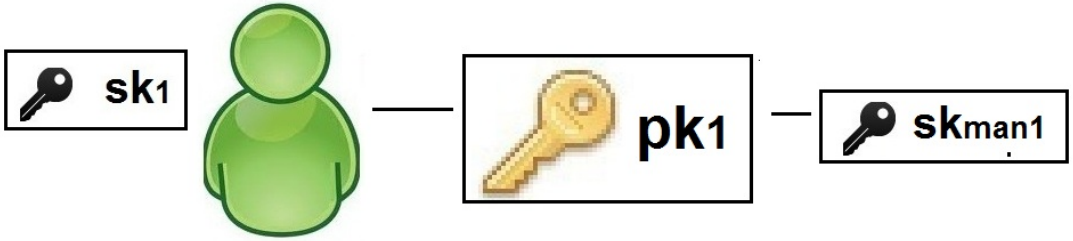
ASGKA

- Introduced by Wu et al. [Eurocrypt'09]
- The parties agree on a same public key
- Each party owns a different secret key



Our Idea

For $n = 2$ ASGKA provides lawful interception abilities...



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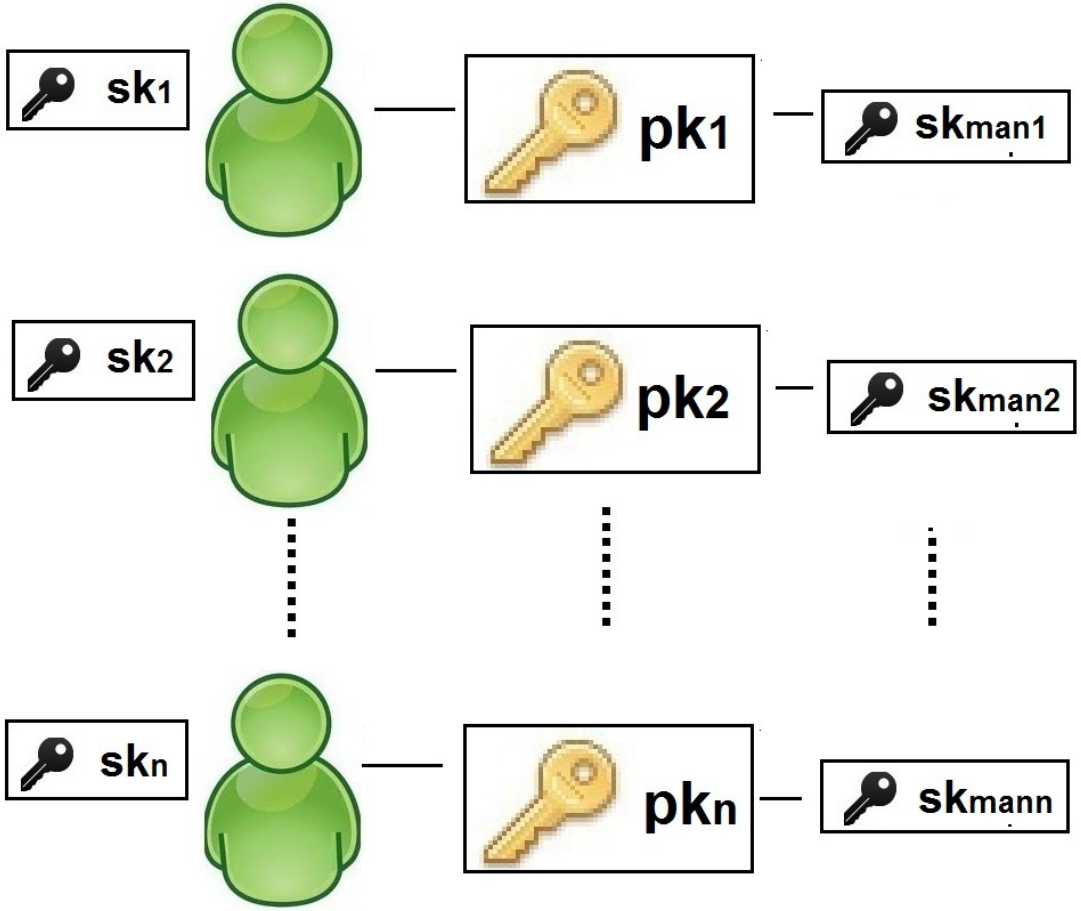
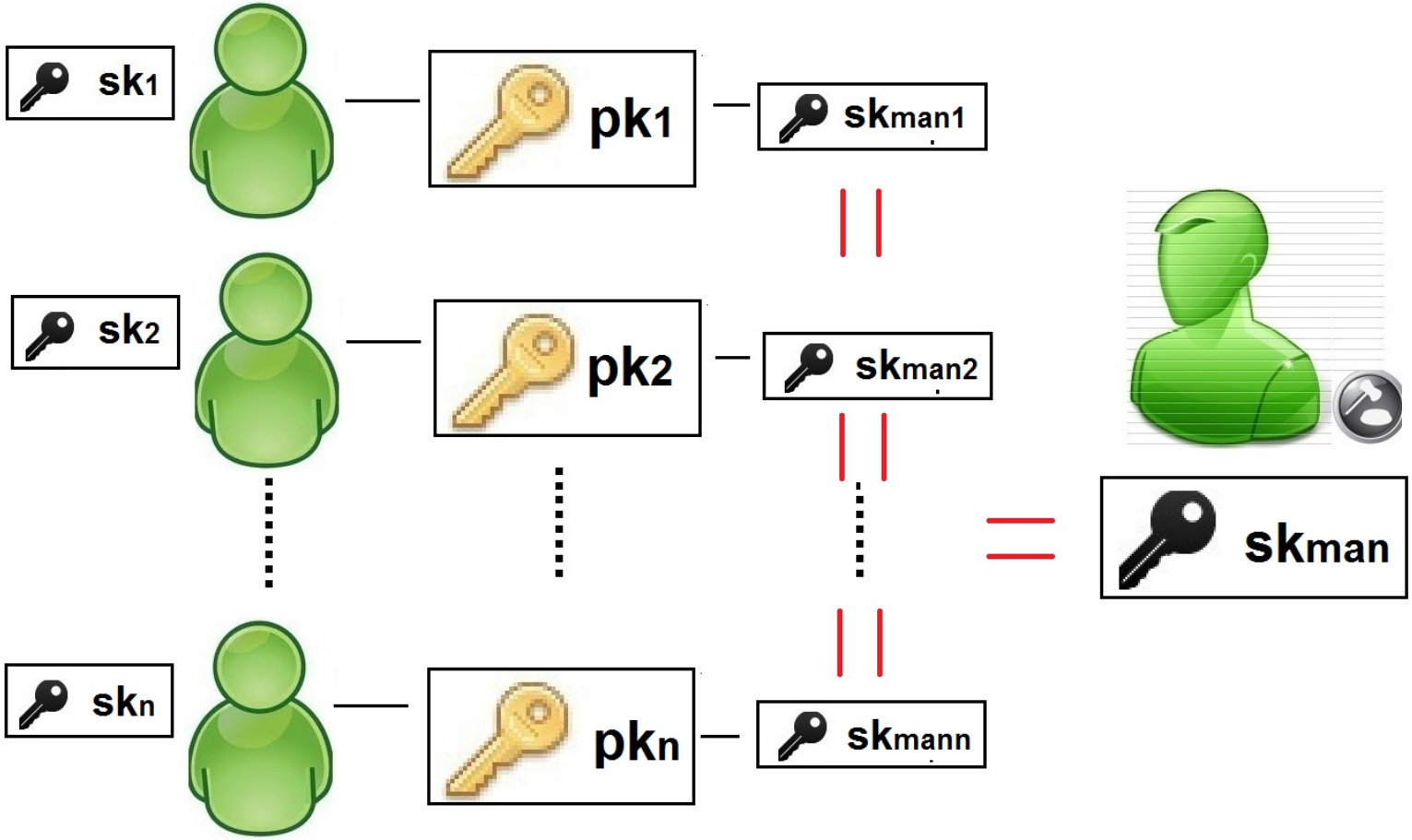
We extend to $n > 2$...



Our Idea

... and make all the manager keys equal.



Preliminaries

Definition

Let G, G_T be 2 cyclic groups of prime order p, $G =< g >$. An efficiently computable bilinear map is a map $e : G \times G \rightarrow G_T$ that satisfies:

1. *Bilinearity:* $e(u^a, v^b) = e(u, v)^{ab}$, $\forall u, v \in G$ and $\forall a, b \in \mathbb{Z}$
2. *Non-degeneracy:* $e(g, g) \neq 1$
3. *Computability:* $e(g_1, g_2)$ *can be efficiently computed*, $\forall g_1, g_2 \in G$

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Decisional Bilinear Diffie-Hellman (DBDH) Assumption

Given g, g^a, g^b, $g^c \in G$, no PPT adversary can distinguish (with a non-negligible advantage) $e(g, g)^{abc}$ from $Z = e(g, g)^z$, $z \in \mathbb{Z}_q$.

Our Proposal

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   $PairGen(1^\lambda) \rightarrow Y = (p, G, G_T, e)$

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   For $h_{man}, X_{man} \leftarrow^R G$, $r_{man} \leftarrow^R \mathbb{Z}_p^*$:
   $R_{man} = g^{-r_{man}}$
   $A_{man} = e(X_{man}, g)$
   $\sigma_{man} = X_{man} h_{man}^{r_{man}}$

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3. **Users keys generation**
   For $i = 1..n$
   $h_i \leftarrow^R G,\ r_i \leftarrow^R \mathbb{Z}_p^*$:
   $X_i = X_{man} h_{man}^{r_{man} - r_i}$
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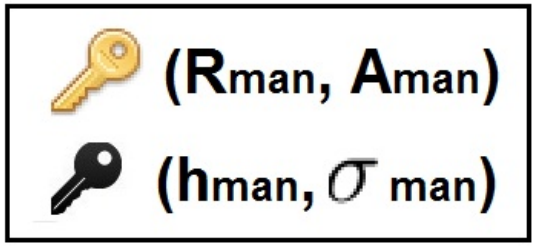

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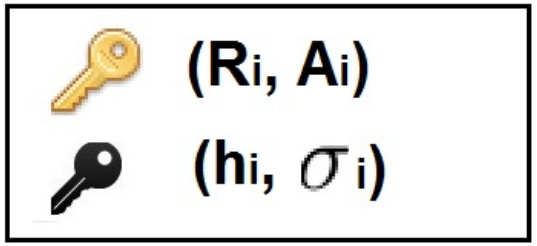

4. **Keys distribution**

Our Proposal

- **Encryption:** A plaintext $m \in G_T$ is encrypted using the public key (R_i, A_i) of a group member P_i as $(t \rightarrow^R \mathbb{Z}_p^*)$:

$$c = (c_1, c_2, c_3) = (g^t, R_i^t, mA_i^t)$$

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  - may be decrypted by P_i:
$$m = \frac{c_3}{e(\sigma_i, c_1)e(h_i, c_2)}$$
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Theorem (Correctness)

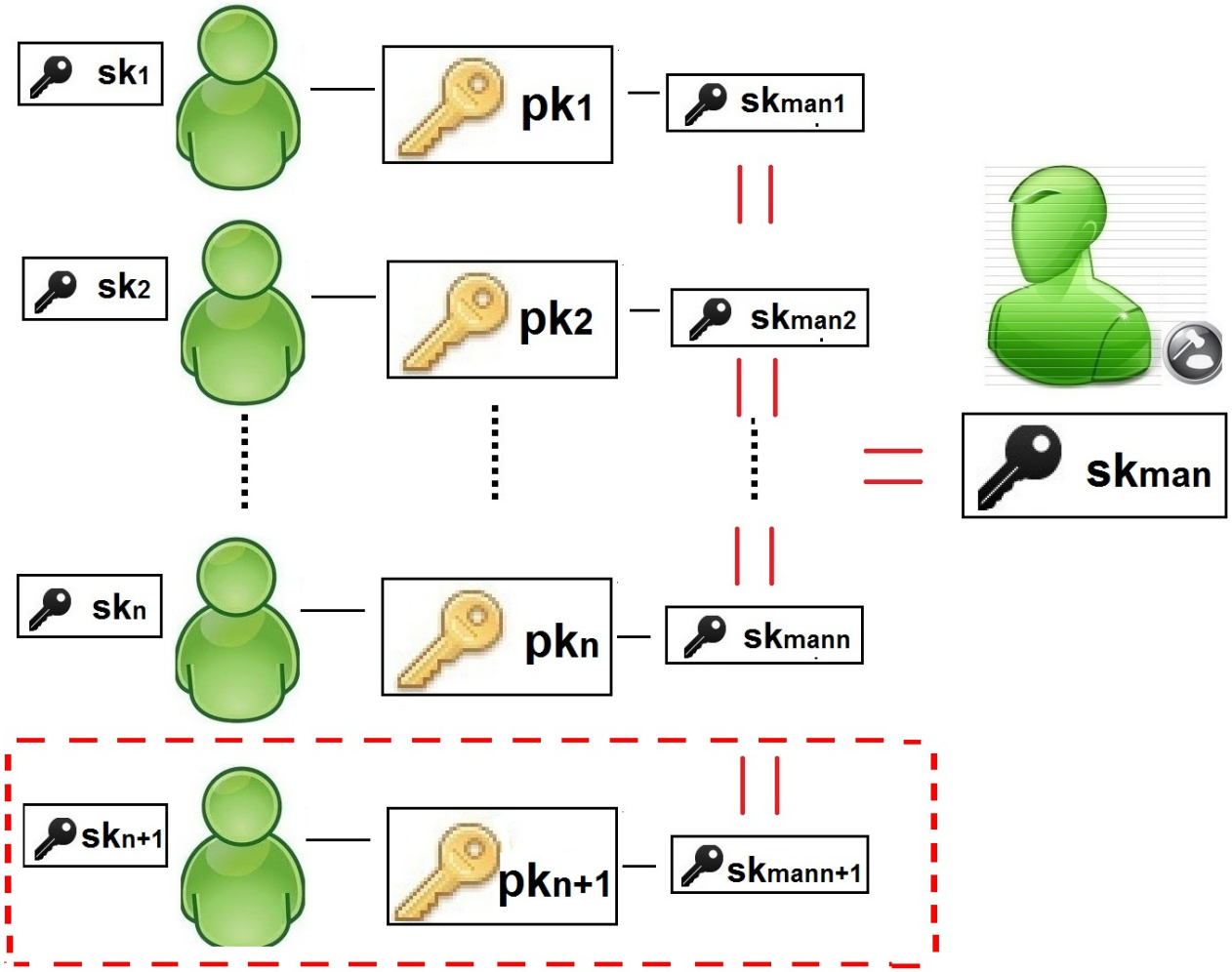
The basic system is correctly defined, in the sense that only the intended group member and the manager are allowed to decrypt a ciphertext.

Ind-CPA Security

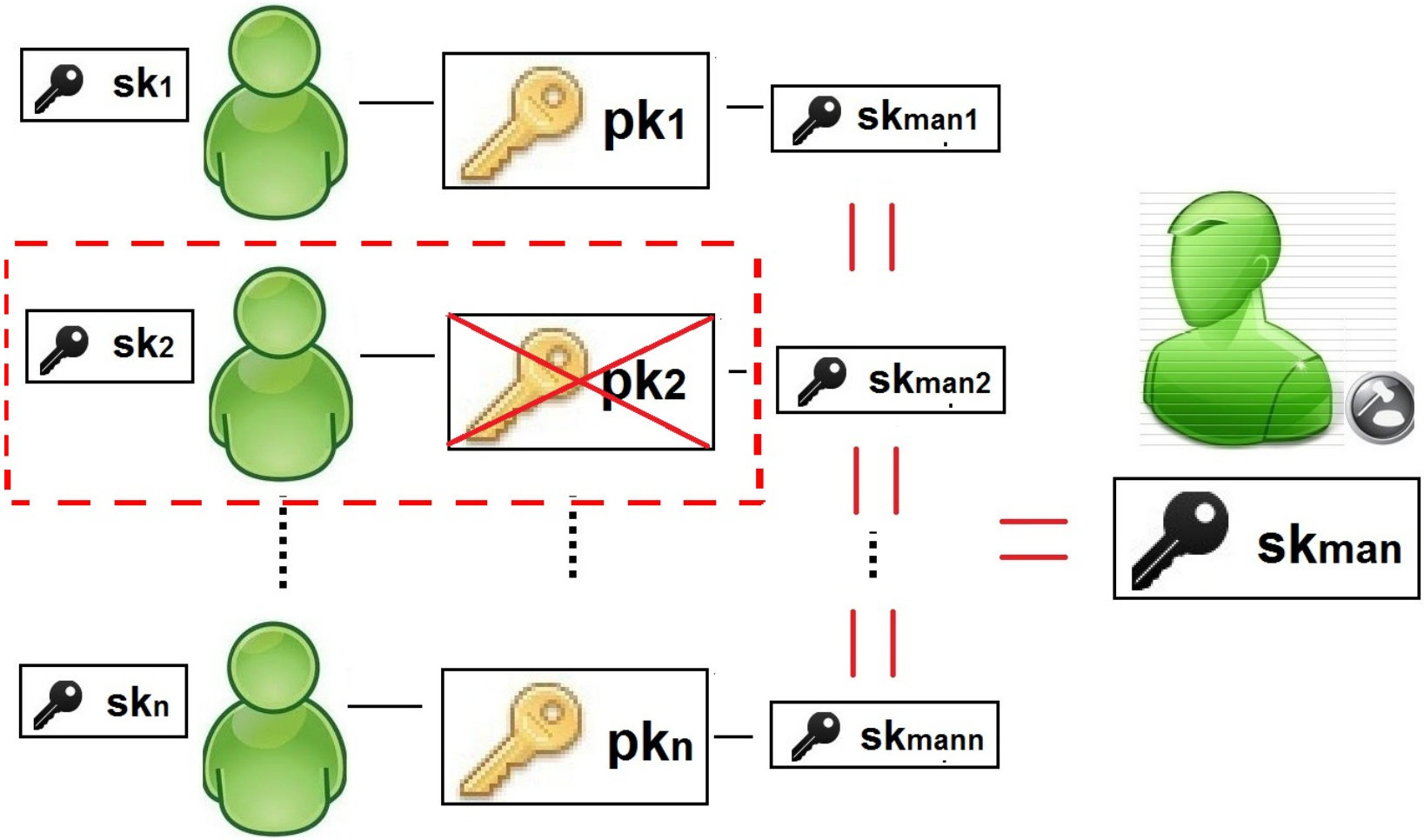
Theorem

The basic system is Ind-CPA secure under the Decisional Bilinear Diffie-Hellman (DBDH) assumption.

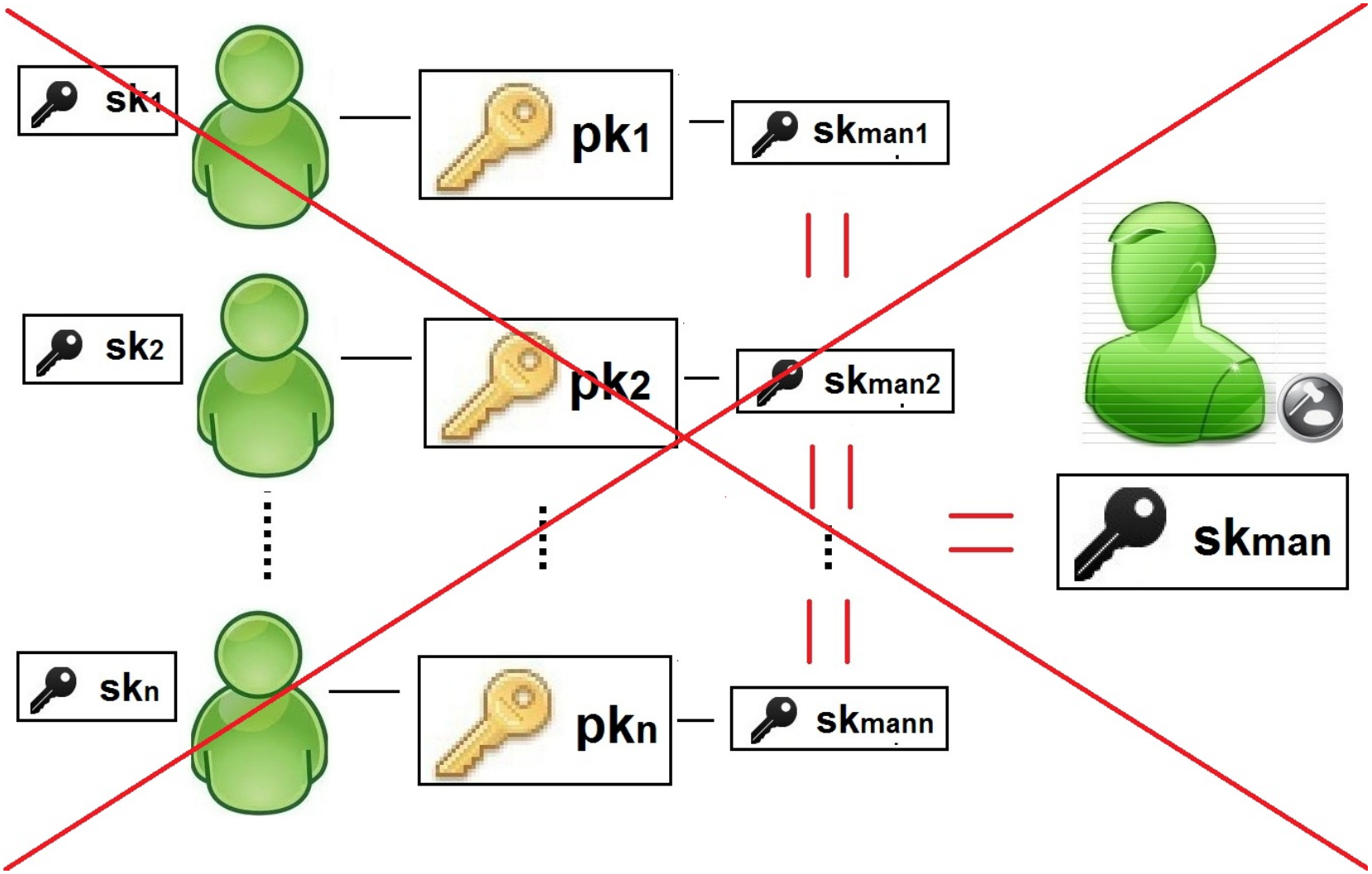
A New Member Joins the Group



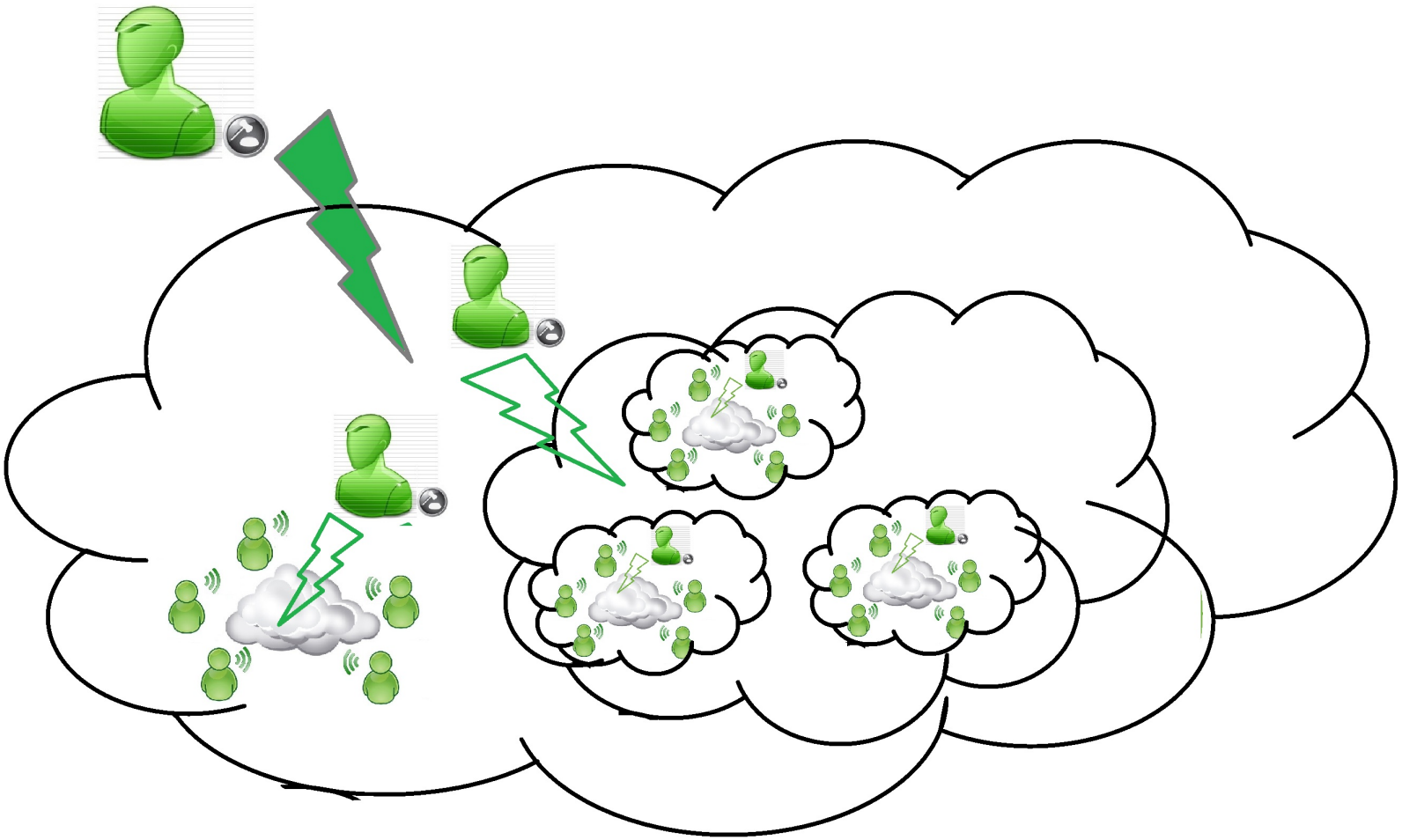
A Member Leaves the Group



The Manager Leaves the Group



Extension to Hierarchical Structures



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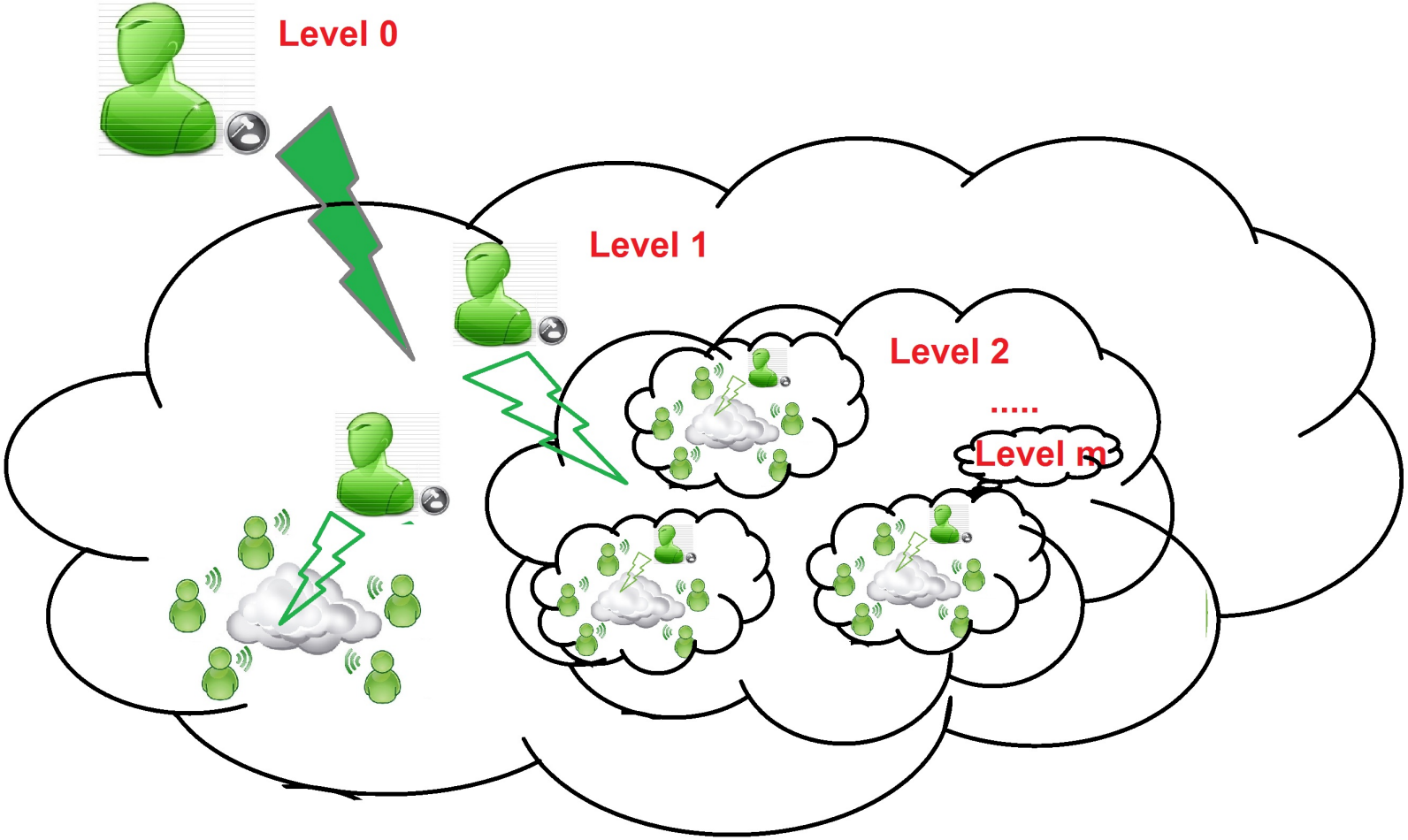

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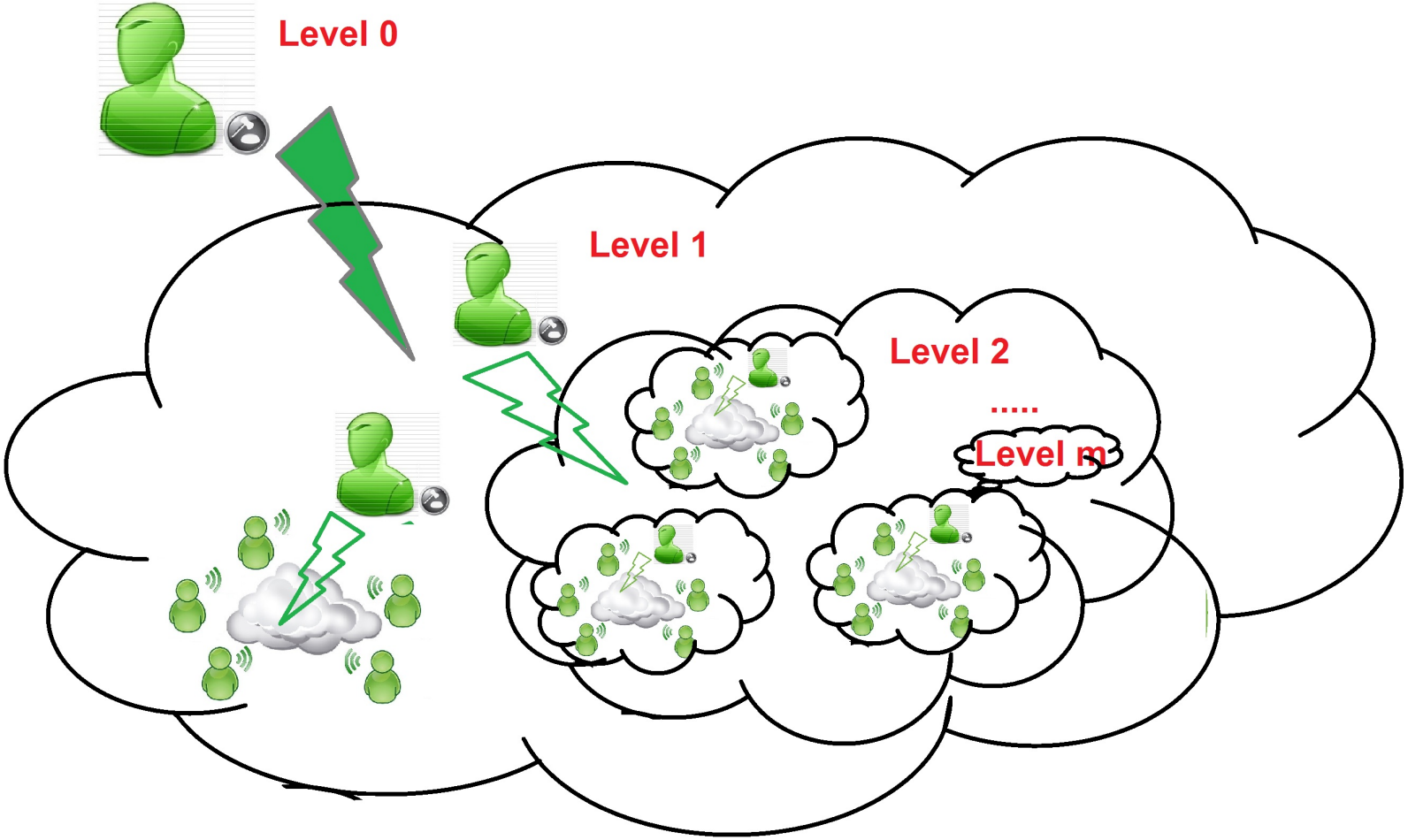
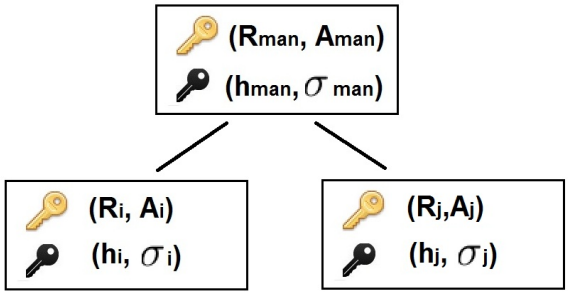


Each secret key and each public key consists of **m** sub-keys

Building Blocks

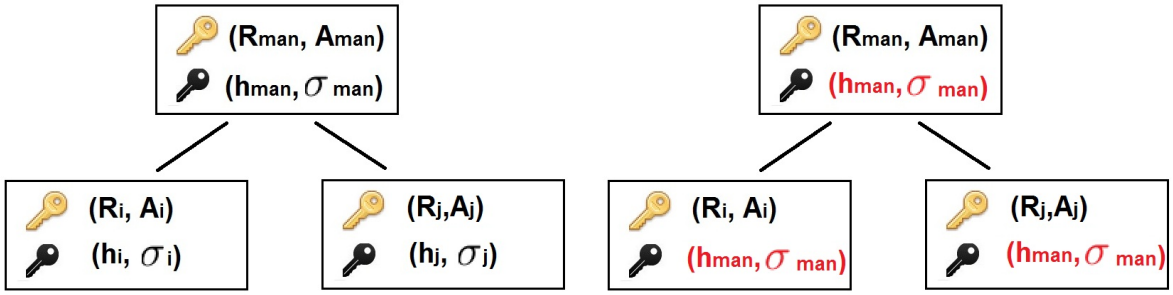
- **BasicGen:** the previously proposed protocol;



BasicGen

Building Blocks

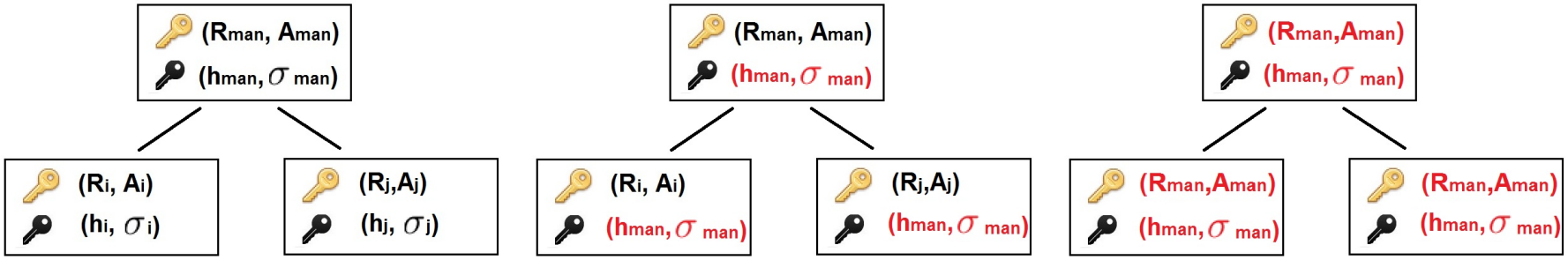
- **BasicGen:** the previously proposed protocol;
- **FwdGen:** BasicGen where $sk_{man} = sk_i$, $\forall i = 1..n$;



BasicGen FwdGen

Building Blocks

- **BasicGen:** the previously proposed protocol;
- **FwdGen:** BasicGen where $sk_{man} = sk_i$, $\forall i = 1..n$;
- **CopyGen:** BasicGen where $sk_{man} = sk_i$ and $pk_{man} = pk_i$, $\forall i = 1..n$;



BasicGen FwdGen CopyGen

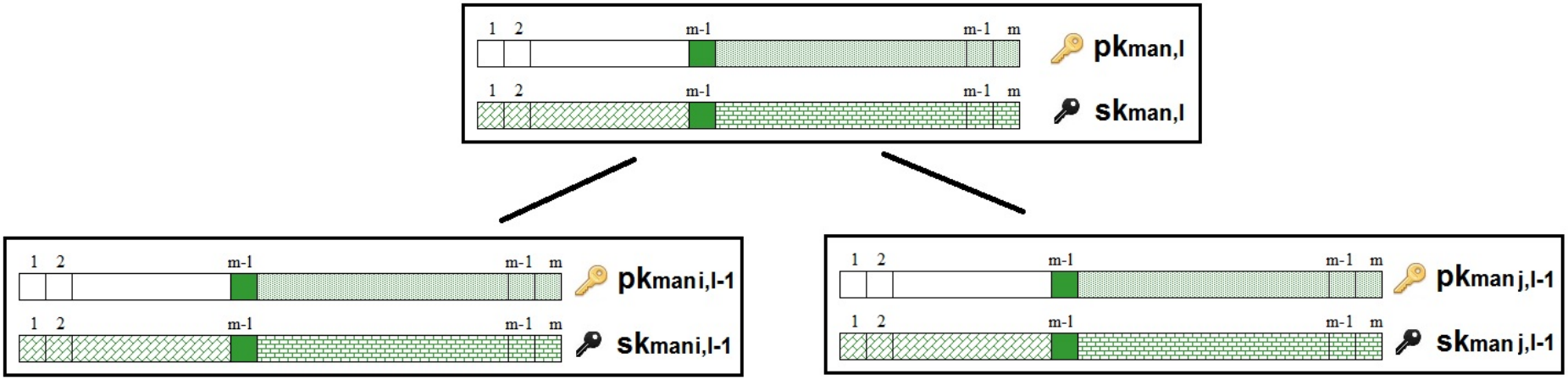
The Extended System

for each level $l = 0..m-1$:

for each participant on level l:

for each key component $i = 1..m$:

- if $i < m - l$: FwdGen
- if $i = m - l$: BasicGen
- if $i > m - l$: CopyGen



The Extended System

- **Encryption:** A plaintext $m \in G_T$ is encrypted using the public key $((R_{i1}, A_{i1}), \ldots, (R_{im}, A_{im}))$ of a group member P_i as $(t \rightarrow^R \mathbb{Z}_p^*)$:
$$c = (c_1, c_{21}, \ldots, c_{2m}, c_3) = (g^t, R_{i1}^t, \ldots, R_{im}^t, m(A_{i1} \ldots A_{im})^t)$$

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$$m = \frac{c_3}{e(\sigma_{i1}, c_1) \ldots (\sigma_{im}, c_1) e(h_{i1}, c_{21}) \ldots e(h_{im}, c_{2m})}$$
  - may be decrypted by P_{man}, any manager of P_i in the hierarchy:
$$m = \frac{c_3}{e(\sigma_{man1}, c_1) \ldots e(\sigma_{manm}, c_1) e(h_{man1}, c_{21}) \ldots e(h_{manm}, c_{2m})}$$

The Extended System

Theorem

The extended system is correctly defined, in the sense that only the intended group member and any of his managers in the hierarchy are allowed to decrypt a ciphertext.

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Theorem

The extended system remains Ind-CPA secure under the Decisional Bilinear Diffie-Hellman (DBDH) assumption.

Future Work

Possible Improvments

- Achieve Ind-CCA security
- Decrease the role of the key generation authority

Thank you!

Questions