Provable Secure Constant-Round Group Key Agreement Protocol based on Secret Sharing

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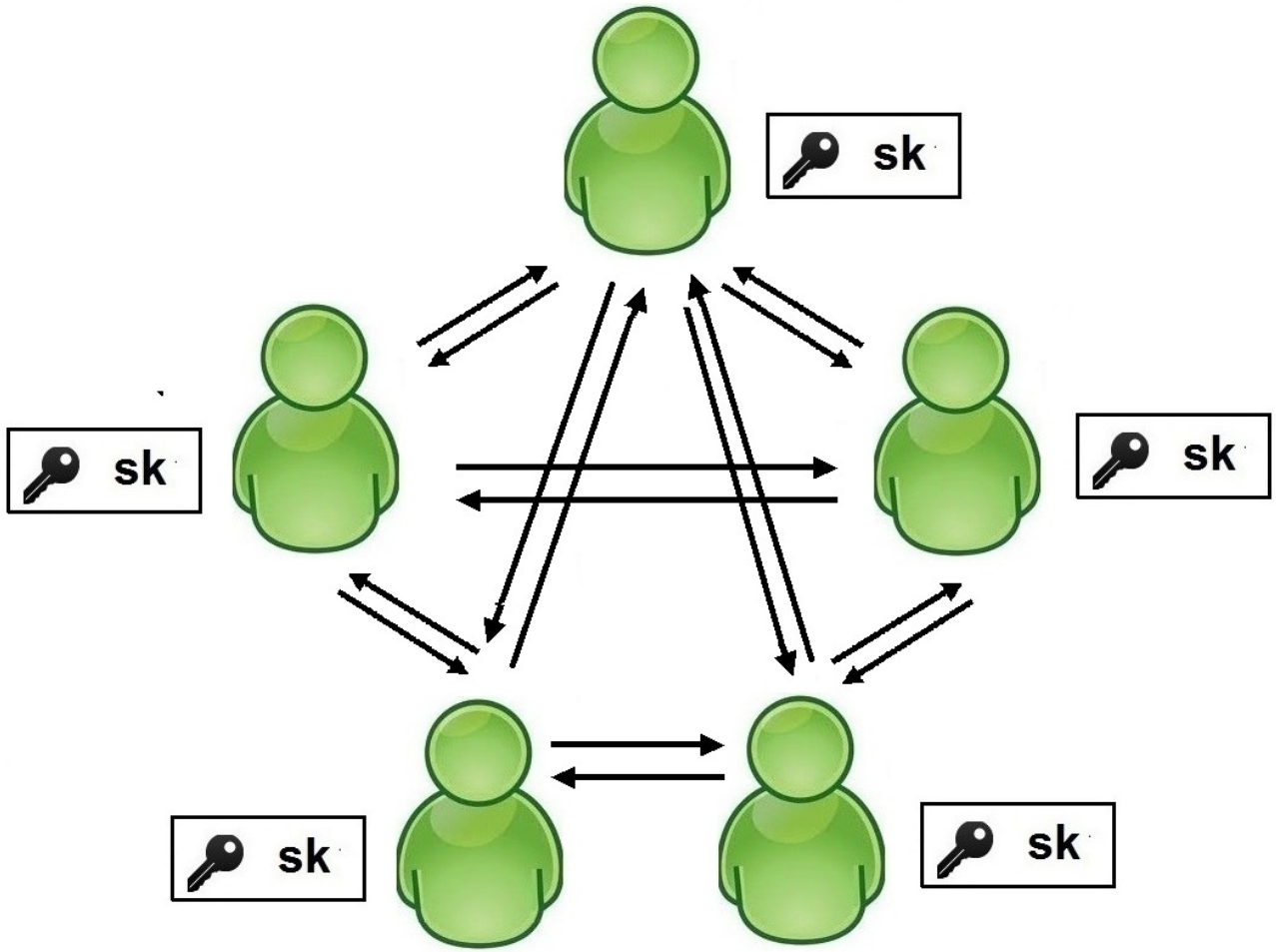
GKA (Group Key Agreement)

Goal: Allow multiple users to share a common secret key for subsequent cryptographic use.



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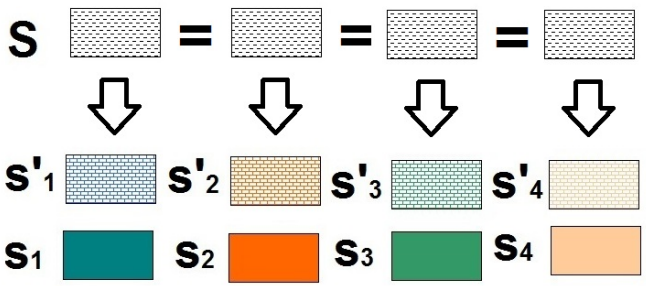
Motivation: few proved secure GKA protocols based on secret sharing exist

Secret n-Sharing



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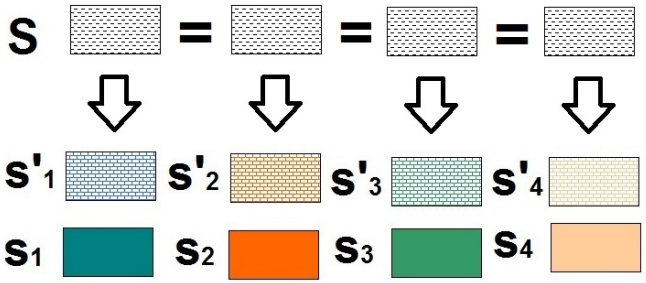
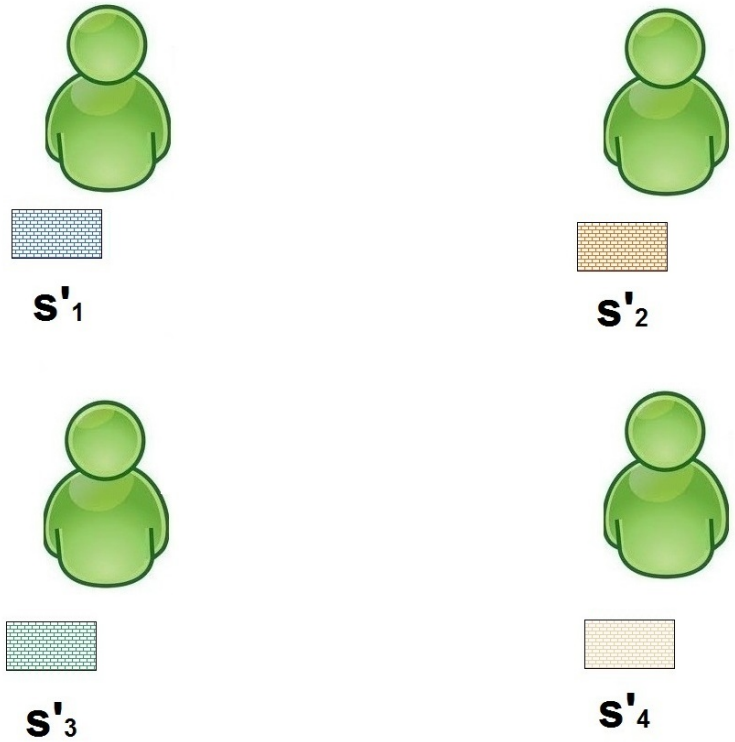
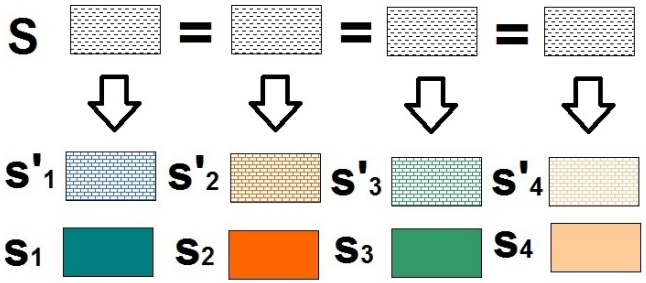
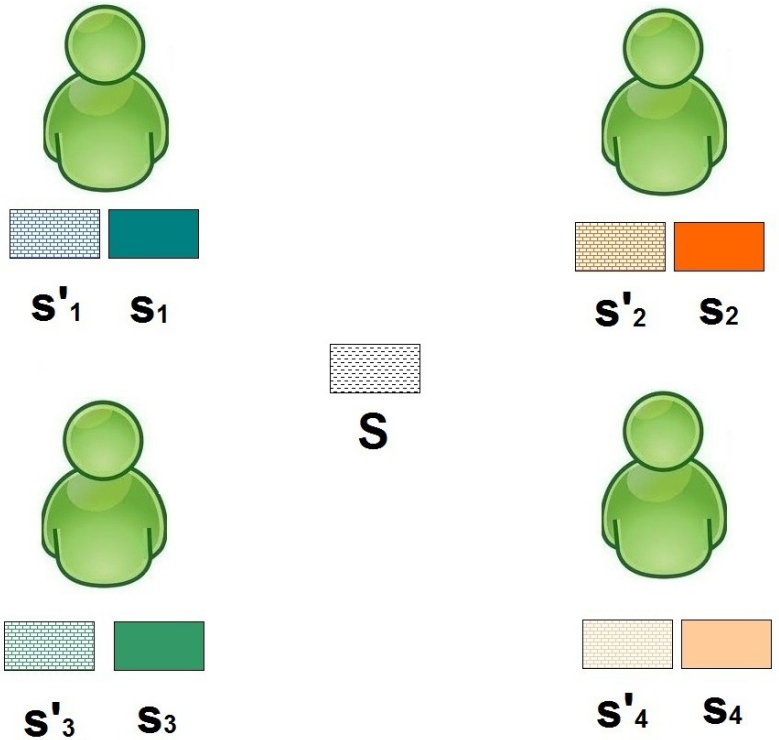




Secret *n*-Sharing





Notations

- The set of users: $\mathcal{U} = \{U_1, \ldots, U_n\}$
- The initiator: U_1
- U_i's signature: $\Sigma = (\mathsf{Sign}_{U_i}, \mathsf{Verify}_{U_i})$. $i = 1 \ldots n$
- A family of trapdoor functions: $\mathcal{F} = (\mathsf{Gen}, \mathsf{F}(\mathsf{pk}, \cdot), \mathsf{F}^{-1}(\mathsf{sk}, \cdot))$
- A secret n-sharing scheme: $\mathcal{SS} = (\mathsf{Share}, \mathsf{Rec})$

Olimid GKA

1. **Round 1.** U_1:
   runs $\mathcal{F}$.Gen to obtain (pk, sk), $r_1 \leftarrow^R \{0,1\}^{\mathcal{K}}$
   $U_1 \rightarrow^*$: $(\mathcal{U}, pk, \mathsf{F}(pk, r_1), \sigma = \Sigma.\mathsf{Sign}_{U_1}(\mathcal{U}||pk||\mathsf{F}(pk, r_1)))$
2. **Round 2.** U_i, $i = 2, \ldots, n$:
   checks that $\Sigma.\mathsf{Verify}_{U_1}(\mathcal{U}||pk||\mathsf{F}(pk, r_1), \sigma) = 1$, $r_i \leftarrow^R \{0,1\}^{\mathcal{K}}$
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   $U_1 \rightarrow^*$: $(r_2', \ldots r_n', \sigma' = \Sigma.\mathsf{Sign}_{U_1}(\mathcal{U}||pk||r_2'|| \ldots ||r_n'||\mathsf{sid}_{U_1}))$
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- Security Notions:
  - AKE: *key confidentiality, forward secrecy, known key security, ...*
  - MA: *mutual authentication, key confirmation, key compromise impersonation resilience, ...*
  - *Contributiveness: equal contribution of the parties to the key establishment, key freshness, key unpredictability, key randomness, ...*

Security Proofs I

Theorem 1 (AKE Security)

If the signature scheme Σ is UF-CMA, the trapdoor function $\mathcal{F}$ is secure, the hash function H is a random oracle and the secret n-sharing scheme $\mathcal{SS}$ is perfect then our protocol is AKE secure and

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{AKE}} \leq 2n^2\mathsf{Adv}_{\mathcal{A},\Sigma}^{\mathsf{UF-CMA}} + \frac{(q_s+q_r)^2}{2^{k-1}} + \frac{(n+1){q_s}^2}{2^{k-1}} + 2\mathsf{Adv}_{\mathcal{A},\mathcal{F}}.$$

Security Proofs II

Theorem 2 (MA Security)

If the signature scheme Σ is UF-CMA and the hash function H *is a random oracle then our protocol is MA secure and*

$$\mathrm{Adv}_{\mathcal{A}}^{\mathrm{MA}} \leq n^2 \mathrm{Adv}_{\mathcal{A},\Sigma}^{\mathrm{UF-CMA}} + \frac{(q_s + q_r)^2}{2^k} + \frac{(n+1){q_s}^2}{2^k}.$$

Theorem 3 (Contributiveness)

If the trapdoor function $\mathcal{F}$ is secure and the hash function H *is a random oracle then our protocol is contributive and*

$$\mathrm{Adv}_{\mathcal{A}}^{\mathrm{Con}} \leq \frac{(n+1){q_s}^2}{2^k} + \frac{q_r}{2^k}.$$

Complexity Analysis

	Storage	Computation	Transmission
U_1	$l_{sk} + l_{sk_1} + k$	$c_{\mathcal{F}.\mathsf{G}} + c_{\mathsf{F}} + (n-1)c_{\mathsf{F}^{-1}} + c_{\mathsf{H}}$ $2c_{\Sigma.\mathsf{Sign}} + (n-1)c_{\Sigma.\mathsf{Verify}}$ $+(n-1)c_{\mathcal{SS}.\mathsf{Share}}$	$l_{\mathcal{U}} + l_{pk} + nl_{\mathsf{F}} +$ $+(n+1)l_{\Sigma.\mathsf{Sign}} +$ $+(n-1)k$
U_i $(i \neq 1)$	$l_{sk_i} + k$	$c_{\mathsf{F}} + c_{\Sigma.\mathsf{Sign}} + nc_{\Sigma.\mathsf{Verify}} + c_{\mathcal{SS}.\mathsf{Rec}}$ $(+(n-2)c_{\mathcal{SS}.\mathsf{Share}})$	

Comparison to existing work

	Bresson-Catalano	Cao et al.	Our Protocol
No.of Rounds	3	3	3
Trans.Type	unicast, broadcast	broadcast	broadcast
No.of Messages	$n(n-1), 2n$	3n	n+1
Group Type	static	static	static
sid Generation	in advance	in advance	at runtime
Security Model	BCP	UC	GBG

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Define new efficient and secure secret sharing-based GKA protocols.

Thank you!

Q & A